Lagrange's Theorem.

The order of each subgroup of a finite group is a divisor (factor) of the order of the group.

Proof:— Let H be a subgroup of a finite group G and let $o(G) = n$, $o(H) = m$.

To prove m is a divisor of n.

For this we have to prove that $n = mp$ for some $p \in H$.

Let Ha be any right coset of H in G.

$o(H) = m \Rightarrow \exists\, m$ distinct elements $h_1, h_2, \ldots h_m \in H$

$\Rightarrow \exists\, m$ distinct elements $h_1a, h_2a, \ldots, h_ma \in Ha$.

for any map from H into Ha is one-one onto.

$\Rightarrow o(Ha) = m = o(H) \;\forall\, a \in G$.

This means that every right coset of H in G has m distinct elements. Since G is finite and hence number of distinct right cosets of H in G will be finite say p.

We also know that any two right cosets of H will be either identical or disjoint. Hence p distinct right cosets of H in G will ~~be either identical~~ contain mp distinct elements.

$\therefore G = H \cup Ha \cup Hb \cup Hc, \ldots$ where $a, b, c, \ldots \in G$.

$o(G) = o(H) + o(Ha) + o(Hb) + \ldots = m + m + \ldots p$ times.

Consequently G will contain exactly mp distinct elements, so that

$o(G) = mp$.

$o(G) = mp \Rightarrow n = mp$. For $o(G) = n$.

Hence the theorem